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APPLICATION OF A MATHEMATICAL MODEL OF ORGANIC MATTER ACCUMULATION AND DECOMPOSITION FOR COMPARATIVE STUDIES OF VARIOUS FOREST FLOOR TYPES*

ABSTRACT: With consideration to the general outline of M i n d e r m a n (1968) a mathematical model of organic matter accumulation and decomposition for three types of forest floor (mull, higromoder and moder-mor) is suggested. Equations determining the kind of functional relations between particular parameters of the model are presented. Empirical data obtained in the preliminary stage of investigations allowed to calculate parameters for each of the three compared types of forest floor and their specific differences as regards the values of these parameters are discussed. The usefulness of the model in characterizing quantitatively the decomposition of organic matter in various types of forest floor is pointed out.

KEY WORDS: Forest ecosystems, forest floor types, accumulation and decomposition of litter, methematical modelling.

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1. INTRODUCTION

In the former paper (P r u s i n k i e w i c z and B i g o s 1978), the results of field investigations on the rhythm of accumulation and decomposition of forest litter in three mixed forest stands (oak, hornbeam, pine) on soils of different forest floor types (mull, higromoder, moder-mor), have been presented in detail. The numerical data were used in graphical diagrams

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to show the flow and accumulation of carbon (CO_2) in the types of forest floor examined. Here, these results will be used to elaborate a mathematical model of organic matter accumulation and decomposition in forest floor layers.

Similarly as in other attempts of mathematical approach to ecological problems, the mathematical model here will fulfil aims as regards: description, analysis, anticipation. The mathematical model is especially useful for:

1. reduction of data, i.e., to limit the vast empirical material to few parameters synthetically characterizing the phenomena and processes analysed;
2. presentation in a precise mathematical form of the phenomena and processes which have been up to now only descriptively expressed;
3. discovering such relations between the phenomena examined which may be difficult to find otherwise;
4. estimating the possibilities for making general conclusions from the results of particular investigations.

Attempts of mathematical approach to quantitative relations between plant fall, its decomposition and accumulation in forest soils have a long tradition. According to Tjurin (1965) this problem has been put up for the first time by P. A. Kostyčev, who in the eightieth years of the nineteenth century expressed the opinion that under particular conditions there has to be an equilibrium between humus accumulation and its losses mainly caused by mineralization. Sibircev (1900–1901) expressed this equilibrium by the following equation:

$$\frac{1}{n}[(n-1)A + A] = A \tag{1}$$

where: A – amount of annual input of organic matter to soil, n – number of years required for total decomposition of A, if assuming identical losses of organic matter each year.

Expression $(n-1)A$ in equation (1) corresponds to the limit of humus store in the steady state. The conception of Sibircev (1900–1901) has two assumptions which do not answer the reality:

1. about identical decomposition rate of fresh and humified plant material;
2. about linear dependence of organic matter losses on time.

Jenny, Gessel and Bingham (1949) have shown that even assuming a constant decomposition rate the disappearance of organic matter in soil cannot be expressed by a straight line but by an exponential curve of the following type:

$$y_t = y_o\, e^{-kt} \tag{2}$$

Many authors (e.g., Olson 1963, Kowal 1969) have assumed such exponential model as a basis for further considerations. And so, Olson (1963) has worked out equations for cases where there was a continuous input of fresh organic matter (coniferous forests) and for cases with discontinuous (discrete) litter fall (deciduous forests shed leavex in autumn). Still, S. Henin, G. Monnier and L. Turc (1959) (after Bunnell, Tait and Flangan 1977) and Tjurin (1965) have mentioned the fact that mineralization rate of fresh plant fall is much higher than that of humified organic residues. Tjurin (1965) has suggested an equation which takes into consideration two rates of decomposition:

$$S = \frac{1 - a}{b} \cdot A \tag{3}$$

where: S – value of limit store of humus in the steady state, a – index of decomposition rate of fresh plant material, b – index of humus decomposition rate ($b < a$), A – amount of annual input of organic matter to soil.

S. Henin, G. Monnier and L. Turc (1959) (after Bunnell, Tait and Flangan 1977) have also assumed two decomposition rates by elaborating a double exponential model. Minderman (1968) has gone even further saying that the division of organic matter into two fractions only (non-humified and humified) is not sufficient and suggested a sum of exponential curves characterizing the decomposition of all main chemical components of plant matter – saccharides, cellulose, hemicelluloses, lignins, waxes and phenols.

Bunnell and Dowing (1974), Bunnell, Tait and Flangan (1977) and Bunnel et al. (1977) have developed the conception of Minderman (1968) taking into account also the effect of temperature and moisture on the decomposition rate of particular components of organic matter. The equation they have suggested is as follows:

$$\frac{dY}{dt} = \sum_{n=1}^{n} \frac{dy_i}{dt} = \sum_{i=1}^{n} - k_i(T, M)\, y_i \tag{4}$$

where Y – the total weight of decomposing substratum, y_i – the weight of substratum component i, k_i (T, M) – the temperature and moisture depending index of decomposition rate of substratum component i.

Replacing the decomposition rate in equation (4) by respiration rate, Bunnell, Tait and Flangan (1977) and Bunnell et al. (1977) have obtained:

$$RR(T, M) = \sum_{i=1}^{n} r_i\ (T, M)\ y_i \tag{5}$$

where: RR (T, M) – total substratum respiration as a function of temperature and moisture, r_i (T, M) – respiration rate of substratum component i as a function of temperature and moisture, y_i – amount of substratum component i.

The suggestion of Minderman (1968) has been used as start point in considerations which are the subject of this paper.

2. CONCEPTION OF THE MODEL

In the forest floor layers of forest soils, usually few (n) subhorizons can be distinguished which differ as to the degree fragmentation and humification of plant fall. In the surface subhorizon AoL there is the dominance of easily decomposable components, whereas successive, deeper subhorizons can be treated as residues of organic matter with increasing decomposition resistance together with the depth. Total store (Y_o) of organic matter accumulated in all forest floor subhorizons at $t = 0$ moment is the sum of store y_{oi} accumulated in particular subhorizons:

$$Y_o = \sum_{i=1}^{n} y_{oi} \tag{6}$$

In each one of the distinguished subhorizons the organic matter disappears[1] with a different mean annual k_i rate, an it is known that this rate decreases together with the depth:

$$k_i > k_2 \ldots > k_n$$

Assuming that at to-day stage of knowledge the exponential function illustrates best the disappearance of organic matter in each subhorizon and assuming for the time being there that is no input of fresh litter, it can be expected that the store of organic matter in subhorizon i of forest floor ($i = 1, 2, \ldots, n$) after time t will be:

$$y_{ti} = y_{oi}\, e^{-k_i t}, \quad (t \geqslant 0) \tag{7}$$

and the total store Y_t will be:

$$Y_t = y_{oi}\, e^{-k_1 t} + y_{o2}\, e^{-k_2 t} \ldots + y_{on}\, e^{-k_n t} \tag{8}$$

which can be also thus expressed:

$$Y_t = \sum_{i=1}^{n} y_{oi}\, e^{-k_i t}, \quad (i = 1, 2, \ldots, n) \tag{9}$$

For an annual period of observation ($t = 1$) equation (7) will take the form:

$$y_{1i} = y_{oi}\, e^{-k_i} \tag{10}$$

Knowing the initial store of organic matter ($t = 0$) and after a year of decomposition ($t = 1$), values k_i can be easily calculated for each subhorizon by transforming the equation (10):

$$k_i = -\ln \frac{y_{1i}}{y_{oi}} = \ln y_{oi} - \ln y_{1i} \tag{11}$$

If we take into consideration that fractions:

$$\frac{y_{o1}}{Y_o} = a_1, \frac{y_{o2}}{Y_o} = a_2, \ldots \frac{Y_{on}}{Y_o} = a_n \tag{12}$$

[1] Disappearance of organic matter in a particular horizon (subhorizon) of soil is mainly due to mineralization and partly to the migration of some decomposition products to other horizons.

are coefficients (weights) showing the contribution of subhorizons 1, 2, ..., n to forming the total Y_o store of forest floor organic matter, then equation (8) can be transformed as follows:

$$Y_t = Y_o(a_1 e^{-k_1 t} + a_2 e^{-k_2 t} \ldots + a_n e^{-k_n t}) \tag{13}$$

and equation (9) as follows:

$$Y_t = Y_o \sum_{i=1}^{n} a_i e^{-k_i t}, \quad (i = 1, 2, \ldots, n) \tag{14}$$

Equation (14) after transformation is as follows:

$$\frac{Y_t}{Y_o} = \sum_{i=1}^{n} a_i e^{-k_i t}, \quad (i = 1, 2, \ldots, n) \tag{15}$$

Fraction $\frac{Y_t}{Y_o}$ defines which part of initial store of organic matter remains in soil after time t. Where as, equation:

$$1 - \frac{Y_t}{Y_o} = 1 - \sum_{i=1}^{n} a_i e^{-k_i t}, (i = 1, 2, \ldots, n) \tag{16}$$

allows to calculate what part of initial store of forest floor organic matter disappears after time t.

Equations (14), (15) and (16) cannot be solved algebraically in relation to t. But (assuming that parameters a_1, a_2, ..., a_n and k_1, k_2, ..., k_n are known) these equations can be used to show the relations:

$$\frac{Y_t}{Y_o} = f(t) \tag{17}$$

or:

$$1 - \frac{Y_t}{Y_o} = g(t) \tag{18}$$

and read from them the course of organic matter decomposition in forest floor layer (Fig. 1). These show especially the time necessary in given conditions for the disappearance of half of the initial store of organic matter in forest floor:

$$0.5 = \sum_{i=1}^{n} a_i e^{-k_i t 0.5}, \quad (i = 1, 2, \ldots, n) \tag{19}$$

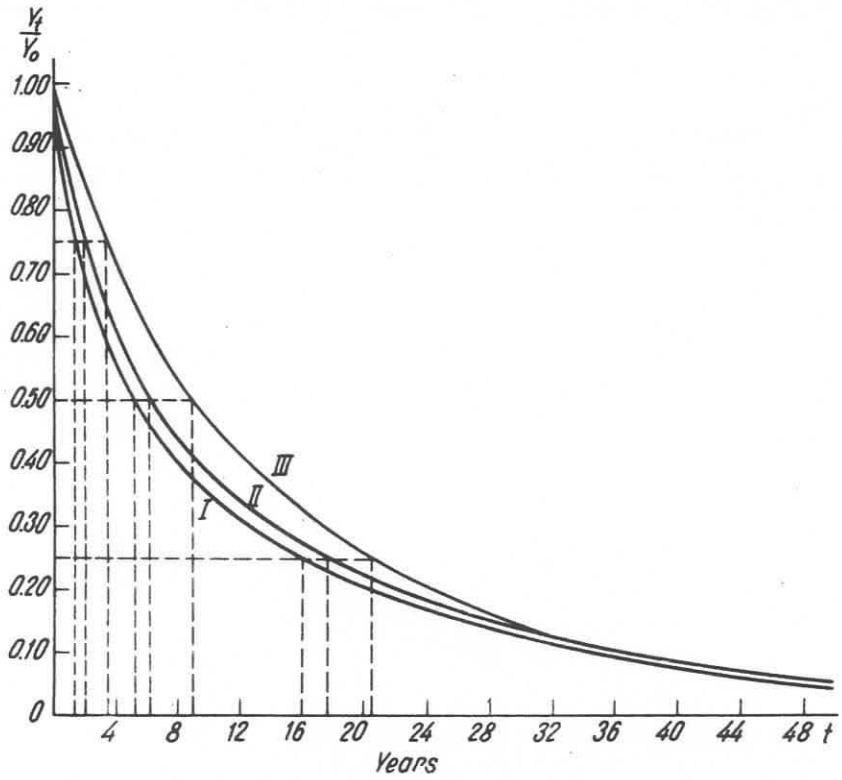

Fig. 1. The organic matter decay curves of various forest floor types (I – mull, II – higromoder, III – moder-mor)
Remaining amounts (Y_t) as related to the organic matter initial stores (Y_o)

Such index may be used to characterize the durability of soil humus, analogously to some extent as the half-life periods for radioelements.

Let's consider now the situation in which relative dynamic equilibrium occurred between decomposition (disappearance) of soil organic matter (D) and the input of fresh litter (L). The values of decomposition and litter fall must be given in comparable units (e.g., in grammes of dry ash free organic matter, in grammes of organic carbon dioxide, or in calories per 1 m^2 of soil surface). Knowing the mean annual litter fall L and mean values of annual indices of disappearance rate of organic matter (k_1, k_2, ..., k_n) in each humus subhorizon, and the contribution of particular subhorizons to the annual disappearance of this matter,

$$\beta_1 + \beta_2 \ldots + \beta_n = \sum_{i=1}^{n} \beta_i = 1, \qquad (i = 1, 2, \ldots, n) \tag{20}$$

and assuming that there is a steady state in the system ($D = L$), the limit (steady) store of soil organic matter ys_i can be calculated for particular subhorizons:

$$ys_1 - ys_1 e^{-k_1} = \beta_1 L$$

$$ys_2 - ys_2 e^{-k_2} = \beta_2 L$$

. . . . . .

$$ys_n - ys_n e^{-k_n} = \beta_n L$$

that is,

$$ys_i (1 - e^{-k_i}) = \beta_i L \tag{21}$$

Thus:

$$ys_i = \frac{\beta_i L}{1 - e^{-k_i}} = \frac{\beta_i D}{1 - e^{-k_i}} \tag{22}$$

Therefore, the limit store of all subhorizons will be:

$$\sum_{i=1}^{n} ys_i = Ys = L \sum_{i=1}^{n} \frac{\beta_i}{1 - e^{-k_i}} = D \sum_{i=1}^{n} \frac{\beta_i}{1 - e^{-k_i}}, \quad (i = 1, 2, ..., n) \tag{23}$$

Frequently coefficients β_i are not known as it may be difficult to measure them. But then the relation between parameters β_i and a_i becomes useful. From the definition (12) it is known that

$$a_i = \frac{ys_i}{Ys}, \quad (i = 1, 2, ..., n)$$

But if instead of ys_i the right side of equation (22) is substituted, then, after appropriate transformations, there is:

$$a_i = \beta_i \frac{L}{Ys (1 - e^{-k_i})} \tag{24}$$

And thus:

$$\beta_i = a_i \frac{Ys (1 - e^{-k_i})}{L} \tag{25}$$

and

$$k_i = \ln (a_i Ys) - \ln (a_i Ys - \beta_i L) \tag{26}$$

The set of equations (24, 25, 26) shows the basic relations existing among parameters which characterize quantitatively the accumulation and disappearance of organic matter in particular humus subhorizons of forest soil. Knowing some of these parameters one may calculate other indices and compare various forest floor types.

3. PARAMETERS OF THE MODEL AND DISCUSSION

For easier comparison in all examined types (subtypes) of forest floor three subhorizons were distinguished ($n = 3$). These subhorizons are described in detail in previous publications (Prusinkiewicz and Biały 1976, Prusinkiewicz and Bigos 1978). According to the results of previous investigations it has been assumed that in forest ecosystems examined which are in the steady state the amount of carbon in annual plant fall is equal to the amount of this element leaving the system over a year as a result of organic matter decomposition ($L = D$).

Table 1. Comparison of parameters of accumulation and decomposition model for three types (subtypes) of forest floor of the reserve "Las Piwnicki"
For explanation of symbols see the text

Parameter	Type (subtype) of forest floor		
	typical mull	higromoder	moder-mor
$L = D$ (g $CO_2 \cdot m^{-2} \cdot year^{-1}$)	876	1,022	967
$ys_1 = y_{o1}$ (g $CO_2 \cdot m^{-2}$)	1,017.82	1,157.46	1,057.91
$ys_2 = y_{o2}$ (g $CO_2 \cdot m^{-2}$)	1,294.55	2,450.65	2,827.77
$ys_3 = y_{o3}$ (g $CO_2 \cdot m^{-2}$)	2,197.25	3,726.51	7,769.37
$Ys = Y_o = \sum_{i=1}^{3} ys_i$ (g $CO_2 \cdot m^{-2}$)	4,509.62	7,334.62	11,655.05
a_1	0.226	0.158	0.091
a_2	0.287	0.334	0.243
a_3	0.487	0.508	0.666
k_1	0.790	0.474	0.243
k_2	0.187	0.190	0.130
k_3	0.046	0.044	0.052
β_1	0.633	0.429	0.252
β_2	0.252	0.413	0.341
β_3	0.115	0.158	0.407
$t_{(0.5)}$ (years)	5.2	6.1	9.0

The comparison of parameters in Table I shows that the forest floors examined differ distinctly. Extreme different properties are displayed by typical mull and moder-mor, whereas higromoder is in between those two, but closer to mull.

Parameters k_i and β_i illustrate well the character of organic matter decomposition of the ecosystems compared. It is interesting that the decomposition rate in deeper subhorizons of forest floor is very similar in the three types compared. Still, basic differences in parameter k are observed in fresh plant fall in the surface layer (AoL). Parameter k_1 of this subhorizon in forest floor of mull type is almost three times higher than in moder-mor. Also, the contribution of particular subhorizons to annual organic matter decomposition (parameter β) is different. In forest floor of mull type over 63% of CO_2 respired from the surface subhorizon, and only 37% from deeper subhorizons. But in moder-mor forest floor type the situation was reverse: surface subhorizon produced over a year hardly 25% of CO_2, whereas the remaining 75% originated from deeper subhorizons.

The time ($t_{0.5}$) required to decompose (disappearance) half of the limit store of organic matter accumulated in forest floor layers of forest ecosystems in steady state is 5.2 years for mull type, 6.1 years for higromoder and 9 years for moder-mor. Diagrams in Figure 1 also indicate the time required for decomposition of 1/4 and 3/4 of total store of organic matter in each forest floor type examined.

With the help of parameters in Table I the relation $Y_t = f(t)$ is presented graphically (Fig. 2) for each type of forest examined. Diagrams demonstrate the decay of organic matter ($g\ CO_2 \cdot m^{-2}$) in particular subhorizons of forest floors and allow to compare them at any time stage.

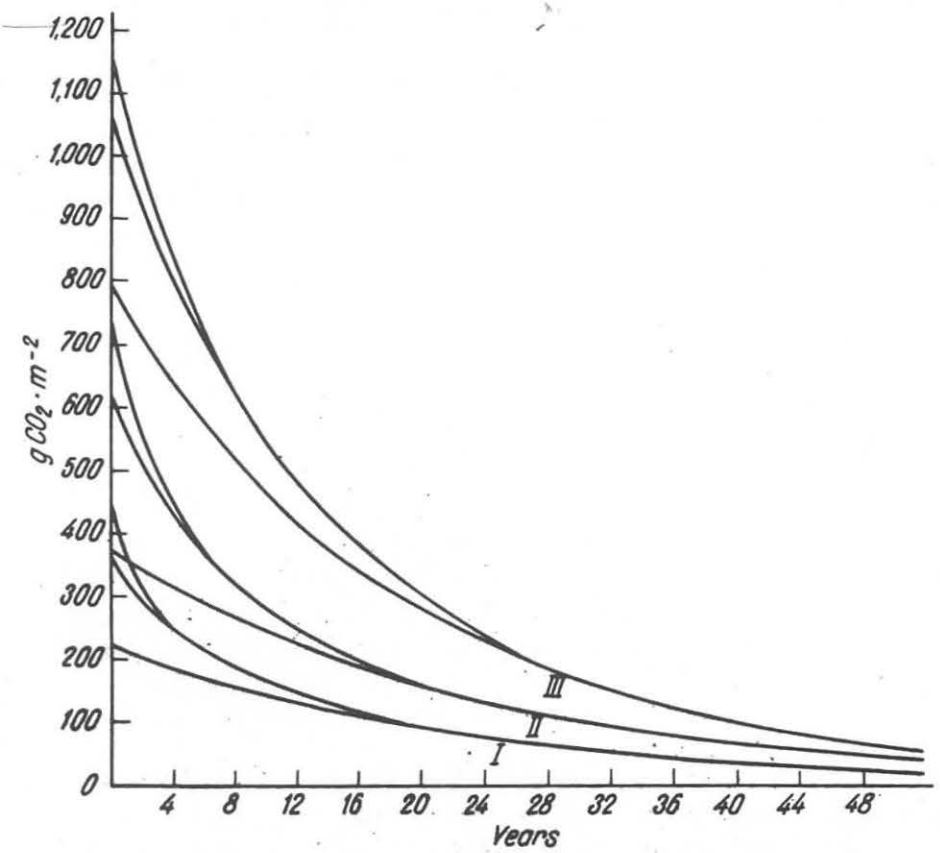

Fig. 2. Diminution of the organic carbon stores (CO_2) in particular forest floor subhorizons as a result of their decay
I – mull, II – higromoder, III – moder-mor

It can be expected that future investigations will confirm the significance of this model also for other forest ecosystems and will prove that there are similar relations between decomposition parameters as those observed for the soil humus of the forest reserve "Las Piwnicki".

4. SUMMARY

The paper presents a brief review of attempts of a mathematical approach to organic matter accumulation and decomposition in forest soils. With consideration to these attempts a development of the Minderman's (1968) conception is presented. This includes also a set of equations determining the kind of functional relations between particular parameters of the model suggested.

The empirical data obtained in earlier field investigations (Prusinkiewicz and Bigos 1978) were used to calculate the numerical values of these parameters (Table I) for three different types of forest floor (mull, higromoder, moder-mor). Also, diagrams illustrating the decomposition (disappearance) of forest floors compared in the function of time were made (Figs. 1, 2).

The usefulness of the model in characterizing quantitatively organic matter decomposition in forest floors of various types is indicated.

5. POLISH SUMMARY

W pracy dokonano krótkiego przeglądu dotychczasowych prób matematycznego ujęcia procesów akumulacji i rozkładu substancji organicznych w glebach leśnych. Na tym tle zaproponowano własne rozwinięcie koncepcji Mindermana (1968). Przedstawiono także zestaw równań określających rodzaj funkcyjnych zależności między poszczególnymi parametrami zaproponowanego modelu.

Korzystając z danych empirycznych uzyskanych we wcześniej prowadzonych badaniach terenowych (Prusinkiewicz i Bigos 1978) obliczono wartości liczbowe parametrów (tab. I) dla trzech różnych typów próchnic leśnych (mull, higromoder, moder-mor). Sporządzono także wykresy ilustrujące procesy rozkładu (zanikania) porównywanych próchnic w funkcji czasu (rys. 1, 2).

Wykazano przydatność modelu dla ilościowej charakterystyki procesów rozkładu materii organicznej w próchnicach leśnych różnych typów.

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